

# TABLE OF CONTENTS

## Part-I & II

### CHAPTER 1: Review of Numbers and Functions

1.1	Real Numbers	1-2
1.2	Complex Numbers	2-3
1.3	Basic Notations	4-11
1.4	Function	11-17
1.5	Basic Functions	17-22
1.6	Graph Shifting	23-27
1.7	Operations with Functions	28-31
1.8	Trigonometric Functions (A brief overview)	32-35
1.9	Chapter Summary	36-39
1.10	Chapter Review	39-40
1.11	Self Test	41-41

**Objective: Graph Shifting**

**New Version** Start with one of the basic graphs and sketch the graph of the following functions.

**Solution**  $f(x) = -x^2 + 1$

**Steps:**

- Select the basic graph for the given function.
- Shift the basic graph using the given arrows.
- Reflect parts of the graph along x-axis using the reflect button.

**Solution :**  
We start with the graph of  $y = x^2$ , reflect the graph in the x-axis, and then lift the resulting graph vertically up 1 unit.

**Discussion**

**Example**

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### CHAPTER 2: Limits and Continuity

2.1	Intuitive Approach	43-44
2.2	Finding Limit “L” Numerically	44-48
2.3	Finding Limit “L” Graphically	49-53
2.4	Finding Limit “L” Algebraically	54-63
2.5	Limit as $x$ Approaches Infinity	64-72
2.6	$(\epsilon, \delta)$ Definition of Limit	73-79
2.7	Continuous Functions	79-84
2.8	Theorems on Continuous Functions	84-88
2.9	Chapter Summary	88-90
2.10	Chapter Review	91-92
2.11	Self Test	93-93

**Objective: Limit (Graphical Approach)**

**Find :**  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

**Solution:**

Graph  $y_1 = \frac{\sin 2x}{x}$  (Figure a)

**Discussion**

**Example**

- Use ZOOM-IN at  $(0, 0)$
- Use TRACE near  $x = 0$ . It shows that  $y$ -values get closer to 1.998 as  $x$  gets closer to 0 from left.
- The  $y$ -values get closer to 1.998 as  $x$  gets closer to 0 from right.

Therefore,  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = 2$

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### CHAPTER 3: The Derivative

3.1	Introduction	95-105
3.2	Derivative of a Function using the Limit Definition	105-115
3.3	Rules for Derivatives	115-138
3.4	Implicit Differentiation	139-144
3.5	Strategy for Differentiation	145-149
3.6	Linear Approximation and Differentials	150-154
3.7	Related Rates	154-160
3.8	Chapter Summary	160-161
3.9	Chapter Review	162-163
3.10	Self Test	163-163

**Objective: Related Rates**

**New Version** A man 6 ft. tall is walking away from a lamp post 15 ft. tall. If the man is walking at a speed of 3 ft/s, how fast is the length of his shadow changing when he is 61 ft. from the lamp post?

**Solution** Rate of change of shadow =  ft/sec

**Solution :**

$6x + 6y = 15y$  Previous steps  $\frac{dx}{dt} = 3$  ft/s

or  $6x = 9y$  or  $y = \frac{2}{3}x$

Take derivative with respect to  $t$  :

$\frac{dy}{dt} = \frac{2}{3} \frac{dx}{dt}$  ----- (i)

We substitute  $\frac{dx}{dt} = 3$  in (i), we get  $\frac{dy}{dt} = \frac{2}{3}(3) = 2$  ft/sec

Therefore, the rate at which the shadow is increasing is 2 ft/sec. ( $\frac{dy}{dt} > 0$ )

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### CHAPTER 4: Applications of the Derivatives

4.1	Introduction	165-165
4.2	Mean Value Theorem	166-170
4.3	Increasing and Decreasing Functions	171-176
4.4	Critical Points	176-179
4.5	Local and Absolute Extrema	180-184
4.6	Concavity	184-190
4.7	Curve Sketching	190-196
4.8	Optimization	196-204
4.9	Antiderivative	205-210
4.10	Chapter Summary	211-212
4.11	Chapter Review	212-213
4.12	Self Test	214-214

**Objective: Optimization**  
 Find the dimensions of a right circular cylinder with maximum volume that can be inscribed in a cone of height 16 in. and base radius 4 in.

**Solution:** Radius  Height  Done

$y = 16 - 4x$  ;  $V = \pi x^2 y$  [Previous steps](#)

**Critical points:**  $x = 0, x = \frac{8}{3}$  [Details](#)

$V = \pi(16x^2 - 4x^3)$

- $V(0) = 0, V(\frac{8}{3}) = \frac{1024\pi}{27}$  and  $V(4) = 0$ .
- The largest volume  $V$  occurs when  $x = \frac{8}{3}$ .
- Therefore, the dimensions of the inscribed cylinder are:  
 Radius  $x = \frac{8}{3}$   
 Height  $y = 16 - 4(\frac{8}{3}) = \frac{16}{3}$

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### CHAPTER 5: Integration

5.1	Approximating Area under a Curve and above the x-axis	215-226
5.2	Definite Integral	227-237
5.3	Theorems of Definite Integral	237-242
5.4	Indefinite Integral (Antiderivatives)	243-249
5.5	More Rules for Indefinite Integrals	250-257
5.6	Strategy for Integration	257-261
5.7	The Fundamental Theorem of Calculus	262-267
5.8	Chapter Summary	267-270
5.9	Chapter Review	270-271
5.10	Self Test	272-272

**Objective: Approximating Area under a Curve and Above the x-axis**

**L-Rule rectangles** **R-Rule rectangles** **Both cases**

**Basic Concept**

**Four L-Rule and Four R-Rule Rectangles**

**Eight L-Rule and Eight R-Rule Rectangles**

**Strategy**

**Summary** (slow) (fast)

By increasing the number of rectangles the area given by the inscribed rectangles increases.

This means that, by increasing the number of rectangles, the area given by the inscribed rectangles gets closer to the actual area.

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### CHAPTER 6: Applications of the Definite Integral

6.1	Area	273-283
6.2	Moments and Center of Mass	284-292
6.3	Volume of a Solid of Revolution	293-302
6.4	Volume by Slicing	303-307
6.5	Formula for Arc Length	307-310
6.6	Surface of Revolution	311-314
6.7	Work	315-321
6.8	Hydrostatic Force	321-324
6.9	Chapter Summary	325-326
6.10	Chapter Review	327-328
6.11	Self Test	329-329

**Objective: Volume of a Solid of Revolution**  
 The region bounded by the x-axis,  $y = \sqrt{a^2 - x^2}$  is rotated about the x-axis. Find the volume of the solid generated.

**Solution:** Enter the answer here Done

**Solution:**  $V = \pi \int_{-a}^a (a^2 - x^2) dx$  [Previous steps](#)

**Step 5:** The limits are clearly from  $-a$  to  $a$ .

**Step 6:** We go back to step 4 and use the limits:

$$\begin{aligned}
 V &= \pi \int_{-a}^a (a^2 - x^2) dx \\
 &= \pi \left[ a^2 x - \frac{x^3}{3} \right]_{-a}^a \\
 &= \pi \left[ \left( a^3 - \frac{a^3}{3} \right) - \left( -a^3 + \frac{a^3}{3} \right) \right] \\
 &= \pi \left( \frac{2a^3}{3} + \frac{2a^3}{3} \right) \\
 &= \frac{4\pi a^3}{3}
 \end{aligned}$$

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# TABLE OF CONTENTS

## Part-I & II

### CHAPTER 7: Calculus of Transcendental Functions

7.1	Basic Results	331-338
7.2	Derivatives of Logarithmic Functions	339-348
7.3	Derivative and Antiderivative of Exponential Function	348-354
7.4	Re-visit the Power Rule and Trigonometric Rules	354-362
7.5	Derivatives of Inverse Trigonometric Functions and Associated Integrals	363-375
7.6	Derivatives and Antiderivatives of Hyperbolic Functions	376-380
7.7	Applications	381-384
7.8	Chapter Summary	385-387
7.9	Chapter Review	388-388
7.10	Self Test	388-388

**Objective: Derivative and Antiderivative of Exponential Function**

**New Version** Find  $\int 3e^{3x+5} dx$

**Solution** Enter the answer here

**Solution :** =  + c **Done**

Recall the standard form:  $\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + C$

We need the *derivative of the power* next to the power function before  $dx$ .

Since  $\frac{d}{dx}(3x+5) = 3$

We write 3 next to  $dx$  to make it *fit into the standard form* for the rule.

Therefore,  $\int 3e^{3x+5} dx = \int e^{3x+5} \cdot 3 dx$

$$= e^{3x+5} + C.$$

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### CHAPTER 8: Techniques of Integration

8.1	Integration by Parts	389-400
8.2	Integrals of $\sin^m x$ or $\cos^n x$	400-407
8.3	Integrals of $\tan^n x$ or $\sec^n x$	408-413
8.4	Integration by Trigonometric Substitution	414-422
8.5	Integration using Partial Fractions	423-430
8.6	More Substitutions	431-435
8.7	Strategy for Integration	435-441
8.8	Numerical Integration	441-450
8.9	Chapter Summary	451-453
8.10	Chapter Review	453-454
8.11	Self Test	454-454

**Objective: Integrals of  $\tan^n x$  or  $\sec^n x$**

**Discussion** **Example 1** **Example 2**

**To integrate  $\tan^n x$  or  $\cot^n x$**

**To integrate  $\sec^n x$  or  $\csc^n x$**

**To integrate  $\tan^n x \sec^m x$  or  $\cot^n x \csc^m x$**

**Evaluate**  $\int \sec^4 x dx$ .

**Solution**

We borrow  $\sec^2 x$  from  $\sec^4 x$  and write it as:

$$\int \sec^4 x dx = \int \sec^2 x \cdot \sec^2 x dx$$

Write the remaining integrand in terms of  $\tan x$ ; use  $\sec^2 x = 1 + \tan^2 x$

$$= \int (1 + \tan^2 x) \cdot \sec^2 x dx$$

$$= \int \sec^2 x dx + \int \tan^2 x \cdot \sec^2 x dx$$

*Prepare for generalized power rule.*

$$= \tan x + \int (\tan x)^2 \cdot \sec^2 x dx$$

*use the generalized power rule*

$$\rightarrow \tan x + \frac{\tan^3 x}{3} + c$$

**Details**

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### CHAPTER 9: Improper Integrals and Polar Coordinates

9.1	Indeterminate Forms	455-462
9.2	Improper Integrals	462-470
9.3	Graphs of Equations in Polar Coordinates	471-480
9.4	Area in Polar Coordinates	481-491
9.5	Parametric Equations	491-495
9.6	Chapter Summary	496-498
9.7	Chapter Review	498-499
9.8	Self Test	499-499

**Objective: Graphs of Polar Equations**

**New Version** Sketch the graph of  $r = 2 \sin 3\theta$ .

**Solution** [Click Here To Select The Answer](#)

**Solution :**

- Test for symmetries :**
  - There is *no symmetry* about the  $x$ -axis.
  - The graph *to be symmetrical* about the  $y$ -axis.
- To find the tangent lines at the pole :**
  - Therefore,  $\theta = 0, \pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3$ .
- Draw Approximate shapes of the curve near the pole :**
- We find a few points to plot on both sides of the tangent line.**
  - Since the graph is *symmetrical* about the vertical axis ( $y$ -axis), we plot points only on the *right side of  $y$ -axis*.

$\theta$	0	$\pi/6$	$\pi/2$
$r$	0	2	-2
- Plot the above points and use the symmetry.

**Note**

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### CHAPTER 10: Infinite Series

10.1 Sequences	501-508
10.2 Bounded and Monotonic Sequences	509-512
10.3 Infinite Series	512-520
10.4 Convergence Tests for a Positive Term Series	520-531
10.5 Alternating Series and Absolute Convergence	532-537
10.6 Power Series	538-542
10.7 Taylor and Maclaurin Series	543-549
10.8 Calculus of Taylor Series	550-554
10.9 Chapter Summary	554-559
10.10 Chapter Review	559-560
10.11 Self Test	560-560

**Objective : Calculus of Taylor Series**

**1** **2** **3**

**Basic properties** Obtain the **Maclaurin series** for  $\ln(1+x)$  for  $|x| < 1$  using

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \dots + \frac{m(m-1)(m-2)\dots(m-k+1)}{k!}x^k + \dots$$

**Solution:**  
We use  $m = -1$ , in the above **Binomial series**, we get

$$f(x) = (1+x)^{-1} = 1 - x + x^2 - x^3 + \dots \quad (|x| < 1)$$

$$\frac{1}{(1+x)} = 1 - x + x^2 - x^3 + \dots$$

Now we **Integrate** both sides.

$$\int \frac{1}{(1+x)} dx = \int 1 dx - \int x dx + \int x^2 dx - \int x^3 dx + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, \text{ for } |x| < 1$$

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### CHAPTER 11: Vectors

11.1 Vectors in Two Dimensions	561-571
11.2 Vectors in Three Dimensions	572-579
11.3 The Dot Product	580-584
11.4 The Vector (Cross) Product	585-591
11.5 Planes in Space	592-596
11.6 Straight Lines in Space	597-602
11.7 Surfaces in Space	603-610
11.8 Chapter Summary	610-613
11.9 Chapter Review	614-614
11.10 Self Test	615-615

**Objective: The Vector (Cross) Product**

**New Version** Find the area of the parallelogram formed by the vectors

**Solution**  $\vec{a} = \vec{i} + 3\vec{j} - 2\vec{k}$  and  $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$

**Solution :**  **Done**

The area of the parallelogram = magnitude of the vector product of  $\vec{a}$  and  $\vec{b}$

First, we find  $\vec{a} \times \vec{b}$ .

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -2 \\ -1 & 2 & 1 \end{vmatrix} = \vec{i}(3+4) - \vec{j}(1-2) + \vec{k}(2+3)$$

$$= 7\vec{i} + \vec{j} + 5\vec{k}$$

Therefore, the **area of the parallelogram** =  $\|\vec{a} \times \vec{b}\| = \sqrt{49 + 1 + 25}$

$$= \sqrt{75} = 5\sqrt{3}$$

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Answers

A.1-A.44

Index

I.1-I.4